

r : Common ratio

What you'll Learn About
 What a geometric series is and whether or not the series converges or diverges
 The nth term test for divergence

$n=0 \quad a_n = a_0(r)^n$
 $n=1 \quad a_n = a_1(r)^{n-1}$

$y = a \cdot b^x$

$y = 4 \cdot 2^x$

$a_n = 4 \cdot 2^n$

Start at $n=0$

Geometric Series
 Converges when

$|r| < 1$

$-1 < r < 1$

$S = \frac{a}{1-r}$

Sum = $\frac{\text{1st term}}{1 - \text{common ratio}}$

Rational Function

Given the first 4 terms of the Geometric Series a) Write the general term of the series, b) Write the power series and c) Find the sum of the series, if possible

A) $4 + 8 + 16 + 32 + \dots + 4(2)^{n-1} + \dots = \sum_{n=1}^{\infty} 4(2)^{n-1} \rightarrow \text{Diverges}$
 (Labels: $n=1$ under 4, $n=2$ under 8, $n=3$ under 16, $n=4$ under 32; "general term" under $4(2)^{n-1}$; "Power series" under the summation)

B) $1 - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} + \dots + 1\left(\frac{-1}{4}\right)^{n-1} + \dots = \sum_{n=1}^{\infty} \left(\frac{-1}{4}\right)^{n-1} = .8$
 (Labels: "converges to" under .8; $r = -\frac{1}{4}$ below the series; $S = \frac{1}{1 - (-\frac{1}{4})} = \left(\frac{4}{5}\right)$ to the right)

C) $5 + 15 + 45 + 135 + \dots + 5(3)^{n-1} + \dots = \sum_{n=1}^{\infty} 5(3)^{n-1} = \text{Diverges}$
 (Label: $r = 3 > 1$ below the series)

D) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots + 1\left(\frac{-1}{2}\right)^{n-1} + \dots = \sum_{n=1}^{\infty} \left(\frac{-1}{2}\right)^{n-1} = \frac{1}{1 - (-\frac{1}{2})} = \frac{2}{3}$

Given the first 4 terms of the Geometric Series; a) Write the general term of the series, b) Write the power series, c) Find the equation for the sum of the series, and d) the Interval and Radius of Convergence of the series.

A) $1 + 5x + 25x^2 + 125x^3 + \dots + 1(5x)^{n-1} + \dots = \sum_{n=1}^{\infty} (5x)^{n-1} = \frac{1}{1-5x}$

Interval of convergence

$$-1 < r < 1$$

$$-1 < \frac{5x}{5} < \frac{1}{5}$$

$$\boxed{-\frac{1}{5} < x < \frac{1}{5}} \text{ I.O.C.}$$

Radius of Convergence = $\frac{1}{5}$

$x = \frac{1}{10}$ $1 + \frac{5}{10} + \frac{25}{100} + \frac{125}{1000} + \dots + 1\left(\frac{5}{10}\right)^{n-1} + \dots = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} = \frac{1}{1-5\left(\frac{1}{10}\right)}$
 signs alternate

~~$1(-x-1)^{x-1}$~~

B) $1 - (x-1) + (x-1)^2 - (x-1)^3 + \dots + (-1)^n(x-1)^n + \dots = \sum_{n=0}^{\infty} (-1)^n(x-1)^n$
 general term power series

$1(-x-1)^n$

$(-1(x-1))^n$

$(-1)^n(x-1)^n$

R.O.C: 1

I.O.C: $-1 < r < 1$

$$\frac{-1}{-1} < \frac{-(x-1)}{-1} < \frac{1}{-1}$$

$$\begin{array}{ccc} 1 > x-1 > -1 \\ +1 & +1 & +1 \\ \hline 2 > x > 0 \end{array}$$

$$= \frac{1}{1 - (-x-1)}$$

$$= \frac{1}{1+x-1}$$

$$= \frac{1}{x}$$